UNSTEADY-STATE FILTRATION OF A LIQUID IN A FRACTURED – POROUS STRATUM TO A WELL WITH A HEMISPHERICAL END-FACE

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A study is made of the motion of a liquid toward a well with a hemispherical end-face with unsteady-state spherical–radial filtration in a fractured–porous stratum consisting of hemispherical regions with different values of the permeability of the system of cracks, superposed one on another. A Laplace transform is used to find exact solutions to the problem of the lowering of the stratum pressure as a function of time and distance as well as of the output of a well working with a constant end-face pressure. The article discusses partial cases corresponding to the exploitation of closed and bounded open fractured–porous strata by a central well with a hemispherical end-face. On the basis of numerical calculations, the effect of the parameters of fractured–porous strata on the change in the indices of the process of their exploitation is established. It is established that, with the exploitation of fractured–porous strata, the process of the lowering of the end-face pressure of the well and its output become stabilized with sufficiently large values of the time.

1. Formulation of Problem. The process of drilling wells in petroleum deposits with fractured–porous types of reservoirs may break down by absorption of the solution right up to the loss of circulation [1]. This makes it impossible to open up a large part of the effective thickness. The ratio of the worked part of the stratum to its total thickness may, in many cases, be assumed to be small, and the flow of liquid toward the well may be assumed to be spherical–radial. It must be postulated that the end-face of the well has a hemispherical form.

To solve the hydrodynamic problems it is postulated that around a well of radius R_* there is a hemispherical region $R_* \le r \le R_0$ with one permeability of the system of cracks and beyond its limits $R_0 \le r \le \infty$, the permeability has another value. A schematic representation of the stratum system is given in Fig. 1.

It is required to determine the process of the lowering of the pressure at an arbitrary point of the hemispherical fractured–porous media superposed one on the other and the output of a well with a hemispherical end-face during the process of exploitation.

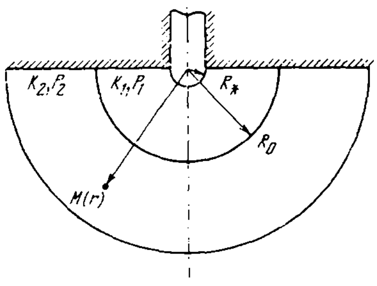

Fig. 1

In accordance with the general theory of the filtration of a homogeneous liquid in a fractured–porous medium [2-4], the basic differential equation with application to a spherical–radial flow can be written in the form

$$\frac{\partial^2 u_i^{(2)}}{\partial \xi^2} + \frac{2}{\xi}\frac{\partial u_i^{(2)}}{\partial \xi} - \frac{1-\omega}{k_i}\frac{\partial u_i^{(1)}}{\partial \tau} - \frac{\omega}{k_i}\frac{\partial u_i^{(2)}}{\partial \tau} = 0 \tag{1.1}$$

$$(1-\omega)\frac{\partial u_i^{(1)}}{\partial \tau} - \lambda\,(u_i^{(2)} - u_i^{(1)}) = 0 \quad (i = 1, 2)$$

$$u_i^{(j)}(\xi, \tau) = p_0 - p_i^{(j)}(\xi, \tau), \quad \xi = r/R_*, \quad \lambda = dR_*^2 k_i^{(1)}/k_i^{(2)}$$

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$$\omega = \frac{m_i^{(2)}\beta_i^{(2)}}{m_i^{(1)}\beta_i^{(1)} + m_i^{(2)}\beta_i^{(2)}}, \quad \tau = \frac{k_1^{(2)} t}{\mu R_c^2 (m_i^{(1)}\beta_i^{(1)} + m_i^{(2)}\beta_i^{(2)})} \tag{1.2}$$

$$k_i = \begin{cases} 1, & i = 1 \\ k_0 = \dfrac{k_2^{(2)}}{k_1^{(2)}}, & i = 2 \end{cases}$$

where α is a coefficient, taking account of the exchange of liquid between the systems of blocks and cracks of the medium; P_0 and $P(\xi, \tau)$ are the initial and instantaneous pressures; k is the coefficient of permeability of the porous medium, μ is the dynamic coefficient of viscosity of the liquid being filtered; m and β are the porosity and the elastic capacity of the stratum; t is the time.

The superscripts 1 and 2 of the functions of the pressure and the parameters of the stratum refer, respectively, to the systems of blocks and cracks of the medium, and the subscripts to the hemispherical regions of the stratum adopted in Fig. 1.

The problem reduces to integration of system (1.1) with the following initial and boundary conditions:

$$u_i^{(1)}(\xi, 0) = u_2^{(2)}(\xi, 0) = 0 \quad (i = 1, 2) \tag{1.3}$$

$$u_1^{(2)}(\xi_0, \tau) = u_2^{(2)}(\xi_0, \tau), \quad \frac{\partial}{\partial \xi} u_1^{(2)}(\xi_0, \tau) = k_0 \frac{\partial}{\partial \xi} u_2^{(2)}(\xi_0, \tau) \tag{1.4}$$

$$u_2^{(1)}(\infty, \tau) = u_2^{(2)}(\infty, \tau) = 0, \quad \xi_0 = R_0 / R_* \tag{1.5}$$

Conditions (1.3)-(1.5) must be supplemented by the assignment of the value of the pressure or the flow rate at the hemispherical surface of the well. In what follows, solutions are given for different working conditions of the well.

2. Determination of the Pressure Field of a Fractured–Porous Stratum with a Constant Output of the Well, Equal to q_0. The condition of the constancy of the output of a well with a hemispherical end-face, with spherical–radial filtration, is written in the form

$$\lim_{\xi \to 1} \left[\xi^2 \frac{\partial}{\partial \xi} u_1^{(2)}(\xi, \tau) \right] = -\frac{\mu q_0}{2\pi k_1^{(2)}} \equiv -Q \tag{2.1}$$

Applying a Laplace transform [5] with respect to the time to systems (1.1) and then eliminating the functions $u_i^{(1)}(\xi, \tau)$ from the transformed systems, taking account of initial conditions (1.3), we obtain

$$\xi^{-2} \frac{d}{d\xi} \left[\xi^2 \frac{d}{d\xi} U_i^{(2)}(\xi, s) \right] - \frac{S}{k_i} U_i^{(2)}(\xi, s) = 0 \quad (i = 1, 2) \tag{2.2}$$

$$S = s[\omega(1-\omega)s + \lambda][(1-\omega)s + \lambda]^{-1} \tag{2.3}$$

The general solution of Eqs. (2.2) has the form [6, 7]

$$U_i^{(2)}(\xi, s) = \frac{A_i}{\xi} \exp(-\xi \sqrt{S/k_i}) + \frac{B_i}{\xi} \exp(\xi \sqrt{S/k_i}) \quad (i = 1, 2) \tag{2.4}$$

where A_i and B_i are integration constants, subject to determination.

In accordance with condition (1.5), for sufficiently large values of ξ, the lowering of the pressure of the external zone of the stratum must revert to zero; it must therefore be assumed that $B_2 = 0$. Finding the remaining constants from conditions (1.4) and (2.1), subject to a Laplace transform, and substituting their expressions into (2.4) the solution of the problem can be represented in the form

$$\frac{U_1^{(2)}(\xi, s)}{Q} = \frac{\xi_0 \sqrt{k_0 S} + k_0 - 1}{\xi s \Phi(S)} \operatorname{sh} \sqrt{S}(\xi_0 - \xi) + \frac{\xi_0 \sqrt{S}}{\xi s \Phi(S)} \operatorname{ch} \sqrt{S}(\xi_0 - \xi) \tag{2.5}$$

$$\frac{U_2^{(2)}(\xi, s)}{Q} = \frac{\xi_0 \sqrt{S}}{\xi s \Phi(S)} \exp[\sqrt{S k_0}(\xi_0 - \xi)] \tag{2.6}$$

$$\Phi(x) = (\xi_0 x + \xi_0 \sqrt{k_0 x} + k_0 - 1) \operatorname{sh} \sqrt{x}(\xi_0 - 1) + [\xi_0 x \sqrt{k_0} + (\xi_0 + k_0 - 1)\sqrt{x}] \operatorname{ch} \sqrt{x}(\xi_0 - 1) \tag{2.7}$$

Since the arguments of the functions entering into the Laplace transforms (2.5) and (2.6) contain a fractional-linear function S determined by expression (2.3), the transition from the transform to the inverse transform is most expediently effected in accordance with the Éfros theorem [5, 8], analogously to [9]. Introducing the notation

$$F_i(\xi, s) = sU_i^{(2)}(\xi, s)\,|_{S=s},\quad g(x, s) = s^{-1}\exp(-xS) \tag{2.8}$$

going over to inverse transforms, and omitting the detailed calculations of the transition around the Bromwich contour [10], we can obtain

$$\frac{u_i^{(2)}(\xi\tau)}{Q} = \frac{2}{\pi\xi\sqrt{k_0}}\int_0^\infty \frac{1-\exp(-u^2\tau/\omega)}{f_1^2(\xi_0, u)+\Omega_1^2(\xi_0, u)} f_i(\xi, u)\frac{du}{u^2} - \frac{2\lambda}{\pi\xi\sqrt{k_0}}\int_0^{\tau/\omega}\varepsilon(\theta)\exp[-b(\tau-\omega\theta)]\,d\theta\int_0^\infty \frac{f_i(\xi, u)\exp(-u^2\theta)}{f_1^2(\xi_0, u)+\Omega_1^2(\xi_0, u)}\,du \tag{2.9}$$

$$f_1(x, u) = \sin u(x-1) + u\cos u(x-1), \qquad b = \lambda(1-\omega)^{-1} \tag{2.10}$$

$$f_2(\xi, u) = \Omega_1(\xi_0, u)\sin u\frac{\xi_0-\xi}{\sqrt{k_0}} - f_1(\xi_0, u)\cos u\frac{\xi_0-\xi}{\sqrt{k_0}} \tag{2.11}$$

$$\Omega_1(\xi_0, u) = \frac{k_0-1-\xi_0 u^2}{\xi_0\sqrt{k_0}u}\sin u(\xi_0-1) + \sqrt{k_0}\cos u(\xi_0-1) \tag{2.12}$$

$$\varepsilon(x) = \int_0^x \exp(-\lambda z) I_0(2\sqrt{b\lambda z(\tau-\omega\theta)})\,dz \qquad (i = 1, 2) \tag{2.13}$$

where $I_0(z)$ is a Bessel function of an imaginary argument, of the first kind and the first order.

The exact solutions of (2.9) obtained describe the process of the redistribution of the dimensionless lowering of the pressure in parts of the inhomogeneous (with respect to permeability) system of cracks of a fractured-porous stratum of hemispherical form, at an arbitrary distance from the surface of the well at any given moment of time.

Substituting $\xi = 1$ into (2.9), we can find the dimensionless lowering of the pressure at the surface of the well in the form

$$u_1^*(1, \tau) \equiv \frac{u_1}{Q} = \frac{2}{\pi\sqrt{k_0}}\int_0^\infty \frac{1-\exp(-u^2\tau/\omega) - u^3V(\tau, u)}{f_1^2(\xi_0, u)+\Omega_1^2(\xi_0, u)}\frac{du}{u^2} \tag{2.14}$$

$$V(\tau, u) = \lambda e^{-b\tau}\int_0^{\tau/\omega}\exp[-(u^2-b\omega)\theta]\,\varepsilon(\theta)\,d\theta \tag{2.15}$$

Formula (2.14) can be simplified for a whole fractured-porous stratum which is homogeneous with respect to the system of cracks ($k_0 = 1$). The denominator of the expression under the integral sign assumes the form

$$f_1^2(\xi_0, u) + \Omega_1^2(\xi_0, u) = 1 + u^2 \tag{2.16}$$

Taking account of relationship 3.446 (I) from [11], we have

$$u_1^*(1, \tau)\big|_{k_0=1} = 1 - e^{\tau/\omega}\operatorname{erfc}(\sqrt{\tau/\omega}) - \frac{2}{\pi}\int_0^\infty V(\tau, u)\frac{u\,du}{1+u^2} \tag{2.17}$$

Assuming that $\omega = 1$, it can be shown that the integral term reverts to zero and that formula (2.17) goes over into the solution for a granular medium [12]

$$u_1^*(1, \tau)\big|_{\omega=1} = 1 - e^{\tau}\operatorname{erfc}(\sqrt{\tau}) \tag{2.18}$$

depending on the time, part of which is tabulated in [13].

It can be shown that, with $k_0 = \xi = 1$, the Laplace transform (2.5) assumes the form

$$U_1/Q = s(1 + S^{1/2})^{-1} \tag{2.19}$$

Expanding the right-hand part of (2.19) in a series and leaving the first two terms in it, for small values of the parameter s (for large values of τ), we obtain

$$U_1/Q \approx 1/s - \sqrt{\omega/s}(s+b/\omega)^{1/2}(s+b)^{-1/2} \doteqdot 1 - \left(\frac{\omega}{\pi\tau}\right)^{1/2}\exp\left(-\frac{b\tau}{\omega}\right)\Big\{1 + \int_0^1 \exp\left(\frac{b\tau-u^2}{\omega}\right) \times$$
$$\times \left[\frac{2-\omega}{\omega} I_0\left(\frac{b\tau u^2}{2}\right) + I_1\left(\frac{b\tau u^2}{2}\right)\right]\exp\left(-\frac{b\tau u^2}{2}\right)du\Big\} \tag{2.20}$$

With $\omega = 1$ (1.20), we have

$$(U_1/Q)_{\omega=1} \approx s^{-1} + s^{-1/2} \doteqdot 1 - (\pi\tau)^{-1/2} \approx u_1^*(1, \tau) \tag{2.21}$$

The asymptotic formula (2.21) describes the process of the lowering of the end-face pressure in a granular medium with sufficiently large values of the time.

The general solutions (2.5) and (2.6) of the given problem make it possible to consider the partial cases $k_0 = 0, \infty$, which are of independent interest.

Let $k_0 = k_2^{(2)} = 0$. This corresponds to the case when a closed fractured-porous stratum of hemispherical form is exploited by a well with a hemispherical end-face with a constant withdrawal of liquid.

The general solutions (2.5) and (2.6) are transformed to the form

$$\lim_{k\to 0} U_i(\xi, s) =$$
$$= \begin{cases} \dfrac{Q}{\xi s}\dfrac{\xi_0\sqrt{S}\,\mathrm{ch}\sqrt{S}(\xi_0-\xi) - \mathrm{sh}\sqrt{S}(\xi_0-\xi)}{(\xi_0 S - 1)\,\mathrm{sh}\sqrt{S}(\xi_0-1) + \sqrt{S}(\xi_0-1)\,\mathrm{ch}\sqrt{S}(\xi_0-1)}, & i = 1 \\ 0, \quad i = 2 \end{cases} \tag{2.22}$$

the inverse transform of which can be represented in the form

$$u_1^*(1,\tau) = \frac{(\xi_0-1)^2 + 3\xi_0}{\frac{1}{3}(\xi_0-1)^4 + \xi_0(2\xi_0^2 - \xi_0 + 2)}\,\frac{\tau/\omega}{\xi_0 - 1} + 2(\xi_0 - 1)\sum_{n=1}^{\infty} d_n^{-2}\frac{d_n^2\xi_0^2 + (\xi_0-1)^2}{d_n^2\xi_0^2 + (\xi_0^2 + \xi_0 + 1)(\xi_0-1)^2} \times$$
$$\times \left\{1 - \exp\left[\frac{-d_n^2\tau}{\omega(\xi_0-1)^2}\right]\right\} - \lambda\int_0^{\tau/\omega}\varepsilon(\theta)f_3(1, \xi_0, \theta)\exp[-b(\tau-\omega\theta)]\,d\theta \tag{2.23}$$

$$f_3(\xi, \xi_0, x) = [(\xi_0-1)^2 + 3\xi_0](\xi_0-1)^{-1}[\tfrac{1}{3}(\xi_0-1)^4 + \xi_0(2\xi_0^2 -$$
$$- \xi_0 + 2)]^{-1} + \frac{2}{\xi}\sum_{n=1}^{\infty}\frac{d_n\xi_0}{\xi_0 - 1}\cos d_n\frac{\xi_0-\xi}{\xi_0-1} - \sin d_n\frac{\xi_0-\xi}{\xi-1}[d_n\xi_0\cos d_n + (\xi_0^2+1)\sin d_n]^{-1}\exp\left[-\frac{d_n^2 x}{(\xi_0-1)^2}\right] \tag{2.24}$$

Here d_n are the roots of the transcendental equation

$$d/\mathrm{tg}\, d - 1 = d^2\xi_0(\xi_0 - 1)^{-2} \tag{2.25}$$

Let $k_0 = k_2^{(2)} = \infty$. This corresponds to the case when, at the external hemispherical surface of a fractured-porous stratum of radius $\xi_0 = R_0/R_c$, there is maintained a constant pressure p_0 over the course of the process of exploitation. The general solutions (2.5) and (2.6) are represented in the form

$$\lim_{k_0\to\infty} U_i^{(2)}(\xi, s) = \begin{cases} \dfrac{1}{\xi s}\dfrac{\mathrm{sh}\sqrt{S}(\xi_0-\xi)}{\mathrm{sh}\sqrt{S}(\xi_0-1) + \sqrt{S}\,\mathrm{ch}\sqrt{S}(\xi_0-1)}, & i = 1 \\ 0, \quad i = 2 \end{cases} \tag{2.26}$$

The inverse transform of (2.26) can be obtained in the form

$$u_1^*(1,\tau) = 2\sum_{n=1}^{\infty}\frac{\xi_0-1}{\beta_n^2 + \xi_0(\xi_0-1)}\left\{1 - \exp\left[\frac{-\beta_n^2\tau/\omega}{(\xi_0-1)^2}\right]\right\} - \lambda\int_0^{\tau/\omega}\varepsilon(\theta)f_4(1,\xi_0,\theta)\exp[-b(\tau-\omega\theta)]\,d\theta \tag{2.27}$$

$$f_4(1, \xi_0, x) = \frac{2}{\xi_0 - 1}\sum_{n=1}^{\infty}\frac{\beta_n^2}{\beta_n^2 + \xi_0(\xi_0-1)}\exp\left[\frac{-\beta_n^2 x}{(\xi_0-1)^2}\right] \tag{2.28}$$

where β_n are the roots of the equation

$$\beta^{-1}\,\mathrm{tg}\,\beta + (\xi_0 - 1)^{-1} = 0 \tag{2.29}$$

The formula corresponding to a granular medium ($\omega = 1$) has the form [12, 14]

$$u_1^*(1,\tau) = 1 - \xi_0^{-1} + 2 \sum_{n=1}^{\infty} \frac{\xi_0 - 1}{\beta_n^2 + \xi_0(\xi_0 - 1)} \exp\left[\frac{-\beta_n^2 \tau}{(\xi_0 - 1)^2}\right] \tag{2.30}$$

3. Determination of the Output of a Well Exploiting a Fractured–Porous Stratum, with a Constant End-Face Pressure. It is postulated that a fractured-porous stratum, inhomogeneous with respect to the permeability of the system of cracks, is exploited by a central well, at whose hemispherical surface a constant pressure, equal to p_*, is maintained.

Replacing conditions (2.1) by

$$p_1^{(2)}(1,\tau) = p_* = \text{const} \tag{3.1}$$

the solution of the problem in a Laplace transform is obtained in the form

$$U_1^{(2)}(\xi, s) = \frac{\sqrt{S}\,\mathrm{ch}\sqrt{S}(\xi_0 - \xi) + ((k_0 - 1)/\xi_0 + k_0\sqrt{S})\,\mathrm{sh}\sqrt{S}(\xi_0 - \xi)}{\xi s \Delta(\xi_0, S)} \tag{3.2}$$

$$U_2^{(2)}(\xi, s) = \frac{\sqrt{S}}{\xi s \Delta(\xi_0 S)} \exp\left[\sqrt{S/k_0}(\xi_0 - \xi)\right] \tag{3.3}$$

$$U_i^{(2)}(\xi, s) \leftrightharpoons u_i^{(2)}(\xi,\tau) \equiv [p_0 - p_i^{(2)}(\xi,\tau)](p_0 - p_*)^{-1} \tag{3.4}$$

$$\Delta(\xi_0, S) = \sqrt{S}\,\mathrm{ch}\sqrt{S}(\xi_0 - 1) + \left(\frac{k_0 - 1}{\xi_0} + \sqrt{k_0 S}\right)\mathrm{sh}\sqrt{S}(\xi_0 - 1) \tag{3.5}$$

By analogy with the introduced notation (2.8), using the Éfros theorem, the inverse transforms of (3.2) and (3.3) can be found in the form [5, 8]

$$u_{1,2}^{(2)}(\xi,\tau) = \varphi_{1,2}(\xi, \xi_0) - \frac{2\sqrt{k_0}}{\pi\xi}\int_0^{\infty} f_{5,6}\left(\xi, \xi_0, \frac{u^2\tau}{\omega}\right)\frac{du}{u^2} - \frac{2\lambda\sqrt{k_0}}{\pi\xi}\int_0^{\tau/\omega} \varepsilon(\theta)\exp[-b(\tau - \omega\theta)]\int_0^{\infty} f_{5,6}(\xi, \xi_0, u^2\theta)\,du\,d\theta \tag{3.6}$$

In formulas (3.6) is adopted the notation

$$f_5(\xi, \xi_0, u^2x) = \frac{u^2 \sin u(\xi - 1)\exp(-u^2x)}{k_0u^2\sin^2 u(\xi_0 - 1) + \Omega_2^2(\xi, u)} \tag{3.7}$$

$$f_6(\xi, \xi_0, u^2x) = \frac{\Omega_2(\xi, u)\sin u(\xi_0 - \xi)/\sqrt{k_0} - u\sqrt{k_0}\sin u(\xi_0 - 1)\cos u(\xi_0 - \xi)/\sqrt{k_0}}{u^{-2}[k_0u^2\sin^2 u(\xi_0 - 1) + \Omega_2^2(\xi, u)]\exp(u^2x)} \tag{3.8}$$

$$\Omega_2(\xi, u) = (k_0 - 1)\xi^{-1}\sin u(\xi_0 - 1) + u\cos u(\xi_0 - 1) \tag{3.9}$$

$$\varphi_1(\xi, \xi_0) = \frac{k_0(\xi_0 - \xi) + \xi}{k_0(\xi_0 - 1) + 1}\xi^{-1}, \quad \varphi_2(\xi, \xi_0) = \frac{\xi_0\xi^{-1}}{k_0(\xi_0 - 1) + 1} \tag{3.10}$$

The formulas obtained, being exact solutions of the problem, describe the process of the lowering of the dimensionless pressure with time at arbitrary points of the internal and external regions of a fractured-porous stratum, exploited by a well with a hemispherical end-face at constant pressure.

Using the expressions for $u_1^{(2)}$ from (3.6), in accordance with a linear Darcy filtration law, the output of the well can be defined in the form

$$Q(\tau) \equiv \frac{\mu q(\tau)}{2\pi k_1^{(2)} R_c(p_0 - p_c)} = \frac{k_0\xi_0}{k_0(\xi_0 - 1) + 1} + \frac{2\sqrt{k_0}}{\pi}\int_0^{\infty} \times$$
$$\times \frac{u^2\exp(-u^2\tau/\omega)\,du}{k_0u^2\sin^2 u(\xi_0 - 1) + \Omega_2^2(\xi, u)} + \frac{2\lambda}{\pi}\int_0^{\tau/\omega} \varepsilon(\theta)\exp[-b(\tau - \omega\theta)]\,d\theta\int_0^{\infty}\frac{u^4\exp(-u^2\theta)\,du}{k_0u^2\sin^2 u(\xi_0 - 1) + \Omega_2^2(\xi, u)} \tag{3.11}$$

In the case of a stratum which is homogeneous with respect to the permeability of the system of cracks, formula (3.11) can be transformed to the form

$$Q(\tau) = 1 + \left(\frac{\omega}{\pi\tau}\right)^{1/2} + \frac{\lambda}{2\sqrt{\pi}}\int_0^{\tau/\omega} \varepsilon(\theta)\exp[-b(\tau - \omega\theta)]\frac{d\theta}{\theta^{3/2}} \tag{3.12}$$

In obtaining formula (3.12) from (3.11), use was made of the relationships 3.321 (2) and 3.461 (2) from [11].

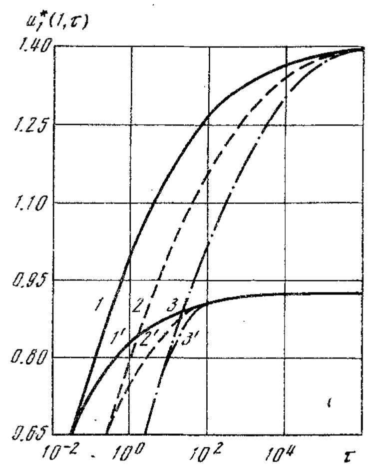

Fig. 2

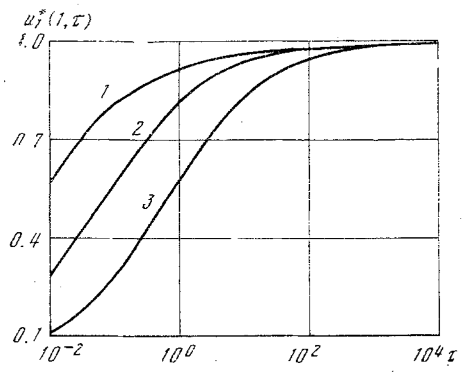

Fig. 3

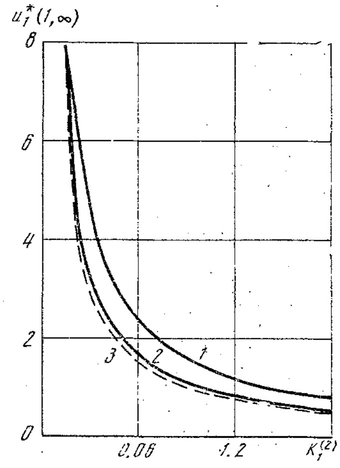

Fig. 4

Solutions (3.6) make it possible to consider partial cases for small and large values of the ratio of the permeabilities of the system of cracks of the stratum.

4. Numerical Calculations. To compare the values of the end-face pressure as a function of time, using exact (2.18) and asymptotic (2.21) formulas for a granular medium, calculations were made, from the results of which it follows that they are in agreement with a rather high degree of accuracy, starting from $\tau = 10$. This is seen from a comparative record: $\tau = 10, 50, 100, 1000, 10{,}000$; $u_1^*(1, \tau) = 0.82927, 0.91943, 0.94139, 0.97888, 0.99267$; $u_1^*(1, \tau) \approx 0.83159, 0.92021, 0.94358, 0.98216, 0.99436$.

The results of calculations made using formula (2.14) for a ratio of the permeabilities $k_0 = 0.2$ and 5, $\lambda = 5 \cdot 10^{-6}$, and $\omega = 1$, 0.1, and 0.01 with $\xi_0 = 10$ are shown by the curves of Fig. 2. Curves 1, 2, and 3 correspond to values of the parameter $\omega = 0.01$, 0.1, and 1 with $k_0 = 0.2$, and curves 1', 2', and 3' to these same values of the parameters ω and λ, but with $k_0 = 5$.

From a comparison of the curves of this figure it follows that, with an increase in the ratio of the permeabilities of the zones of a fractured–porous stratum k_0, there is a decrease in the values of the function $u_1^*(1, \tau)$ corresponding to exactly the same value of the time.

The curves of Fig. 3 were plotted from calculations using formulas (2.17) and (2.18) with $k_0 = 1$ and $\lambda = 0.005$. Curves 1, 2, and 3 correspond to values of the parameter of the cracking capacity ω equal, respectively, to 0.01, 0.1, and 1. It can be seen that, with a decrease in the value of the parameter ω, the function $u_1^*(1, \tau)$ increases and then becomes adjacent to the curve for a granular medium.

The curves of Figs. 2 and 3 approach their asymptotes, since the process of the lowering of the end-face pressure is stabilized with a sufficiently large value of the time τ. This is characteristic also for a granular medium [12, 14].

The value of the function of the lowering of the pressure depends on the value of $k_1^{(2)}$. The form of this dependence is shown in Fig. 4 for the case of the fully established state $\tau \to \infty$. Curves 1 and 3 were plotted for the case $\xi_0 = 10$, and curve 2 for $\xi_0 = 500$. The solid lines correspond to the value $k_0 = 0.2$, and the dotted line to $k_0 = 5$.

We give below the results of calculations using formula (3.12), determining the value of the dimensionless output of a well with a hemispherical end-face, with time, for different values of the parameters ω and λ:

τ	0.001	0.005	0.01	0.1	1	10
$Q\binom{\omega = 0.1}{\lambda = 5 \cdot 10^6}$	2.80	2.08	1.80	1.30	1.16	1.10
$Q\binom{\omega = 0.01}{\lambda = 0.005}$	3.18	2.48	2.25	1.76	1.36	1.15
$Q\binom{\omega = 0.1}{\lambda = 0.005}$	6.64	3.61	3.31	2.15	1.45	1.16
$Q\binom{\omega = 1}{\lambda = \infty}$	18.84	8.98	6.64	2.78	1.56	1.18

It follows from these data that, in granular and fractured–porous strata, the change in the output of a well with a hemispherical end-face is stabilized after the lapse of a definite interval of time in the process of exploitation. Depending on the values of the parameters ω and λ, the outputs of wells differ considerably with a fixed value of τ. With an increase in the parameter λ as well as with the approach of the value of ω to unity, the value of the output of a well in a fractured–porous stratum approaches the value for a granular medium.

Analogous conclusions can be drawn for fractured–porous strata with an inhomogeneous permeability of the system of cracks $k_0 \neq 1$.

The process of the stabilization of the lowering of the end-face pressure of a well with a hemispherical end-face and its output with the unsteady-state spherical–radial filtration of a homogeneous liquid in fractured–porous media is not characteristic for other forms of flows (plane–parallel and plane–radial), as is borne out by the results of investigations [9, 15-17].

The values of the functions entering into the calculating formulas were taken from [18-20].

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